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Technical Notes

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Mixture Optimization of Superdetonative Ram Accelerator Using Refined Response Surface Method

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Introduction

RAM accelerators, which exploit shock-induced combustion around a projectile, were first introduced by Hertzberg et al.[1] in the late 1980s as a new device to accelerate payload to hypervelocity. Such devices can be classified into two types according to their operating modes: thermally choked mode and super-detonative mode. Through several experimental and numerical investigations, the effect of mixture compositions (oxidizer, propellants, and dilute gas) on the ram accelerator performance has been investigated, and the existence of optimal mixture compositions has been verified.[2] Recently, numerical optimization techniques were implemented to improve the ram accelerator performance. For a ring-mounted projectile operating between Mach 6.4 and 8.0, Sabean and Lewis[3] performed a shape optimization by selecting projectile configuration as the design variable and thrust coefficient as the design objective. They approximated the thrust coefficient as a quadratic function of flight speed by evaluating the thrust at several flight conditions.

In this study, having investigated the acceleration characteristics of the ram accelerator at the superdetonative mode, the optimization problem will be formulated to determine the optimal mixture composition of the ram accelerator. To reduce efficiently the significant computational cost during design and optimization stages, a refined system approximation technique utilizing the response surface method (RSM) is proposed and applied. The refined RSM utilizes the design space-transformation and stretching technique to improve the accuracy of system approximation near the optimum region where the highly nonlinear gradient dominates. By the comparison of the optimization results with those obtained from the direct optimization that does not use any approximation, the efficiency of the method is demonstrated.

Numerical Analysis of Ram Accelerator Flowfield

The ram accelerator undergoes nonequilibrium chemical reactions at high pressures and temperatures. The flow inside the ram tube is assumed to be inviscid to improve the computational efficiency. Two-dimensional reactive Euler equations are selected as the governing equations. Roe's flux difference splitting scheme with monotone upstream-centered schemes for conservation laws (MUSCL) is implemented for the spatial discretization for higher-order extension, and a fully implicit lower–upper symmetric Gauss–Seidel scheme is employed for time integration (see Ref. 4). To account for nonequilibrium chemical reaction for mixtures involving hydrogen, oxygen, and nitrogen, Moretti's eight-step and seven-species chemical reactions model[5] is employed. A wedge–cylinder–wedge configuration, a generic configuration in a superdetonative mode, is selected as a baseline configuration. The half-angle of the wedge is 14 deg, and the inner radius of the acceleration tube R is 2 cm. The radius and the total length of the projectile are $0.75R$ and $10R$, respectively. The mass $m = 100$ g and geometry of the projectile are fixed during the optimization process. To investigate numerical convergence, several grid systems, 130×22, 200×30, and 280×40, are examined, Results indicate that the 130×22 grid with moderate stretching near the midportion of the projectile provides reasonable solutions and is adopted in the present study. The total force and the acceleration exerted on the projectile can be easily calculated by summing up the axial components of the projectile surface pressure.

Mixture Composition Optimization

The optimum mixture composition that minimizes the ram tube length, that is, which quantity provides the best acceleration characteristics, are examined by implementing several optimization techniques. Emphasis will be given to the reduction of the computational time and cost. The ram tube is initially filled a mixture of $H_2/O_2/N_2$ at 20 atm and 300 K. The initial projectile velocity V_0 is set to be 2500 m/s and the exit velocity V_e is 3000 m/s. The ram tube length is selected as an objective function in the problem, and the relative mole ratios of H_2 and N_2 to O_2 are selected as the design variables X_1 and X_2, respectively. Side constraints are specified to define the design space area, $2.0 \leq H_2 \leq 2.5$ and $3.7 \leq N_2 \leq 5.0$. The equivalence ratio $\Phi \geq 1$ is included as a design constraint to prevent ignition failure caused by insufficient heat release. The optimization algorithms implemented in this study are a gradient-based sequential linear programming (SLP) method[6] and a genetic algorithm.[7]

The thrust coefficient is the measure of an acceleration force normalized by the initial pressure force of the mixture and represents the performance of a ram accelerator. The ram tube length, which is selected as the objective function, can be obtained by integrating the thrust coefficient or the acceleration from the initial velocity V_o to the exit velocity V_e. The acceleration performance is evaluated by approximating the thrust coefficient C_t as a quadratic polynomial of the projectile velocity.[3] Three different velocities, 2500, 2750, and

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3000 m/s, from V_o to V_e are selected to evaluate C_t at each mixture composition.

Refined RSM for the System Approximation

To reduce computational loads, the RSM,[8] a popular system approximation technique originated from the design of experiments,[8] is considered. Emphasis is given to increasing the confidence level, that is, R^2_{adj}, an accuracy criterion of the system approximation, which is expressed as a ratio of the total deviation to the responses of the experiment points. R^2_{adj} has values in between 0 and 1. If R^2_{adj} of a system is above 0.9, this model can be regarded as a reliable model. Once the regression model is constructed, the optimal design point can be obtained using either a gradient-based method or a genetic algorithm.

With a second-order response function, it is very difficult to represent the highly nonlinear objective function and the design constraints properly. To increase the accuracy of the response function, or to construct a reliable regression model that can best approximate the highly nonlinear real world phenomena, to increase the order of the polynomial might be the easiest way. By the increase of the order, not only the required number of the experiment points (or analysis runs) increases exponentially, but also a reliable approximation is not possible when there exists a high gradient near the optimum. Therefore, an efficient design is not possible in many practical design problems.

To approximate the real behavior of the objective function more accurately, the region near the optimum point must be emphasized. One way to overcome the nonlinear behavior of the objective function is to transform the design space using stretching functions. In other words, a weighing is given to the region where the optimum point possibly exists. No restrictions exist for the selection of the stretching functions. Any functions that can specify the stretching location and stretching amount can be employed. In this study, following function is used.[9] In the following equation, D_i is the range of the design space and ζ_i and x_i are the natural variable and the coded variable, respectively. The clustering parameter β_i and the clustering position C_i are the control parameters for the design space transformation,

$$x_i = B_i + (1/\beta_i)\sinh^{-1}[(\zeta_i/C_i - 1)\sinh(\beta_i B_i)]$$

where

$$B_i = \frac{1}{2\beta_i}\ln\left[\frac{1 + (e^{\beta_i} - 1)(C_i/D_i)}{1 + (e^{-\beta_i} - 1)(C_i/D_i)}\right]$$

Results and Discussion

By the implementation of gradient-based SLP, the optimum mixture composition has been obtained after seven design iterations, which results in the reduction of 19% of the launch tube length. The initial design starts from the infeasible region, $X_1 = 2$, $X_2 = 5$, and $\Phi = 0.8$, to verify the robustness of the optimization procedure.

Application of Conventional RSM

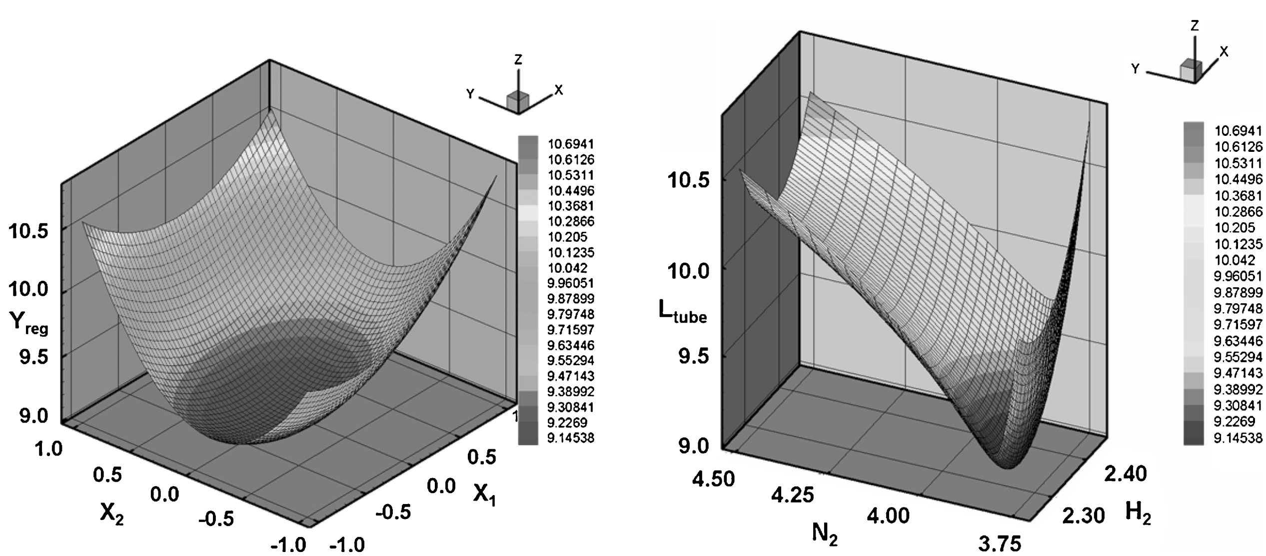
To alleviate the computation cost, the system approximation technique utilizing the RSM has been implemented. To construct the regression model 15 points are used, and the optimized ram tube length is 9.77 m, 8.1% bigger than that of the direct optimization (Table 1). Although the number of analysis runs has been reduced when compared with previous gradient-based optimization, which requires 21 points, a considerable difference with the analysis result is observed with the value of adjusted R-square R^2_{adj} of 0.92. Again, this is caused by the approximation of the highly nonlinear behavior of ram accelerator acceleration characteristics with second-order polynomial.

Application of Refined RSM

Because the regression model and the optimization result of the conventional RSM are not satisfactory, improvement on the accuracy of the system approximation is necessary. As already mentioned, a refined RSM using the design space stretching technique is devised and implemented to approximate the highly nonlinear behavior of the response with second-order polynomials.

Table 1 Comparison of the optimization results

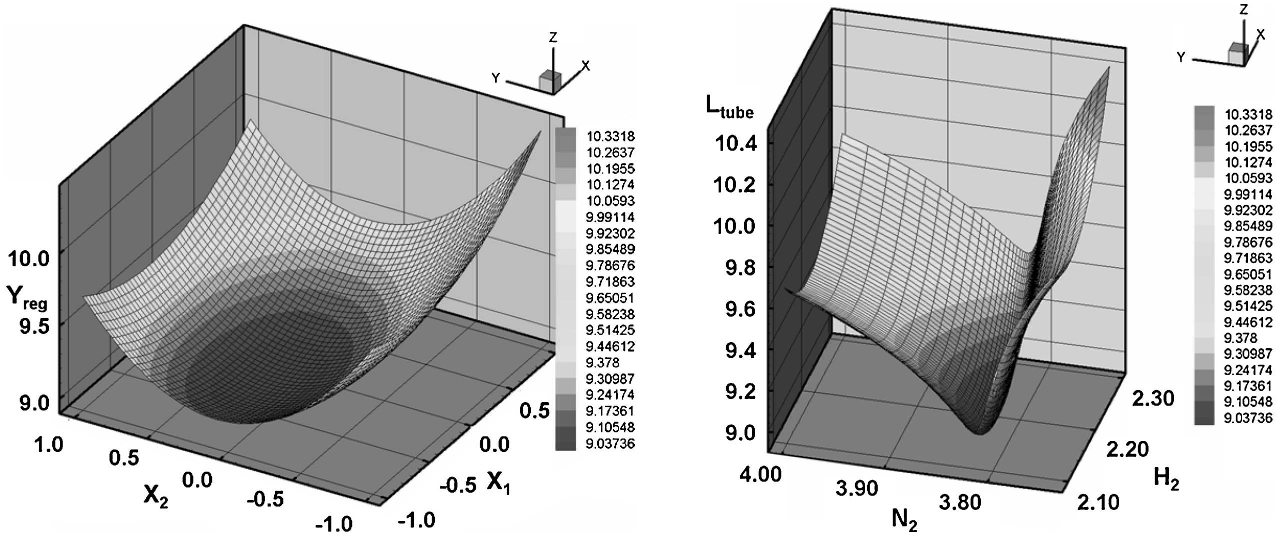
Optimization method	β	Cluster position	Mole no. (H_2, N_2)	Confidence level	L_{tube}	Error, %	No. of analysis runs	Improvement from initial design, %
Direct optimization	——	——	(2.16, 3.81)	——	9.04	——	21	19
Conventional RSM	——	——	(2.20, 3.95)	0.92	9.77	8.1	15	12.5
Refined RSM	5	0.30	(2.16, 3.82)	0.97	8.97	0.77	14	19.6
Refined RSM	7	0.27	(2.15, 3.85)	0.98	9.05	0.11	15	19



a) Response in coded variable space

b) Response in natural variable space

Fig. 1 Response surface model with design space stretching, $\beta = 5$.



a) Response in coded variable space

b) Response in natural variable space

Fig. 2 Response surface model with design space Stretching, $\beta = 7$.

In the mixture composition optimization, it is noticed that the acceleration characteristics are more sensitive to the N_2 composition rather than the H_2. Hence, design space is stretched in the N_2 direction. Stretching parameter β of 5 and 7 are used in the construction of response function. The results are summarized in Table 1 and Figs. 1 and 2. The confidence level R^2_{adj} is found to be 0.97 for $\beta = 5$ and 0.98 for $\beta = 7$, respectively. With good regression models, the optimal values are very close to the result obtained by the gradient-based optimization method. With stretching the design space only in the N_2 direction, the RSM shows a remarkable improvement. For the same design space, the gradient-based method requires 21 points to find the optimum, but the refined RSM requires only 14 points. For all cases considered in this study, RSM requires fewer number of experiment points than direct optimization method.

As can be seen in Fig. 1, the second-order response function in transformed space can accurately approximate the stiff variation of the objective function in the natural space. As a result, the optimal ram tube length is very close to the gradient-based direct optimization results.

Table 1 compares the results of the conventional and refined RSM with results from direct optimization. When refined RSM is applied, the location of the optimum point becomes more accurate and the maximum error with respect to the numerical calculation falls in the range of $1 \sim 3\%$ by constructing reliable regression models. Moreover, the percent deviations from the direct optimization case are less than 1%, which shows considerable improvement in comparison with the deviation of 8.1% in the conventional RSM case.

The method developed can be directly applied to more general nonlinear problems with many design variables. To increase the efficiency, the design space is better to be quickly narrowed down using direct optimization techniques. The RSM can then reduce the number of design analyses greatly with appropriate stretching design space.

Conclusions

The effect of mixture composition on the performance of a ram accelerator has been identified by investigating the pressure distribution on the projectile surface and the location of the detonation wave. The mixture composition was successfully optimized using the gradient-based SLP technique. The ram tube length was reduced by 19%. Implementations of the refined RSM during the design optimization lead to a global optimum solution with 50% fewer analysis runs. When the natural design variables were stretched, the highly nonlinear behavior of the objective function was approximated accurately with a quadratic polynomial of design variables, and hence, an improved optimum solution was obtained.

Acknowledgment

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